

Mixing and lifetime of B-meson using Coulomb plus power potential

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Abstract

The investigation of mixing phenomena and lifetime in neutral B meson systems provides an important testing ground for standard model flavour dynamics. Spectroscopic parameters has been used to calculate the pseudoscalar decay constant and predictions of mixing mass parameters and lifetimes of the B_d and B_s mesons.

1 Introduction

The study of mixing and lifetimes of the B -meson, provides us useful information about the dynamics of quarks and gluons at the hadronic scale. The remarkable progress at the experimental side, with various high energy machines such as LHC, B-factories, Tevatron, ARGUS collaborations, CLEO, CDF, DO *etc.* for the study of hadrons has opened up new challenges in the theoretical understanding of B-meson. In order to understand the structure of the newly observed zoo of open flavour meson resonances[1, 2, 3, 4] in the energy range of 2-5 GeV, it is necessary to analyze their spectroscopic properties and decay modes based on theoretical models. Many of these states could be the excited beauty mesonic states while for many other states the possibility of multi-quark or molecular like structures are being proposed. Thus, the main objective of the present paper includes the study of decay constant, mixing parameters and lifetimes of the open flavour beauty mesons using the spectroscopic parameters. We study these open beauty mesons within the frame work of a potential model [5, 6]. The weak decay constants of pseudoscalar and vector decay constants are important in the calculations of various decay rates [7] and mixing parameters of the mesons.

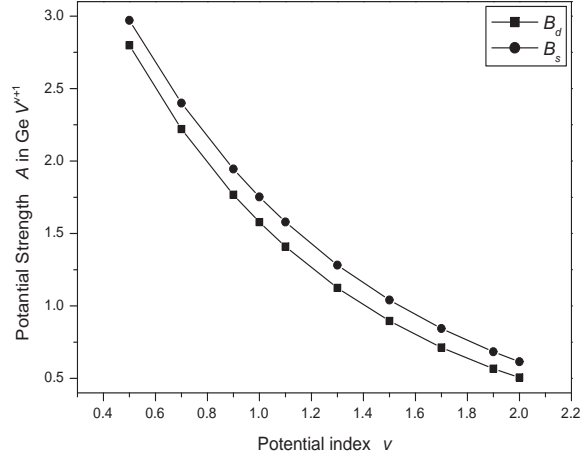


Figure 1: Behavior of A with the potential index ν for B_d and B_s mesons. GeV

2 Theoretical methodology: A Potential Scheme

In the limit of heavy quark mass $m_Q \rightarrow \infty$, heavy meson properties are governed by the dynamics of the light quark. As such, these states become hydrogen like atom of hadron physics. We investigate the heavy-light mass spectrum of B and B_s mesons in the frame work of a relativistic potential model. For the light-heavy flavour bound system of $q\bar{Q}$ or $\bar{q}Q$, we treat the heavy-quark ($Q = b$) as well as the light-quark ($q = d, s$) relativistically within a potential bound mesonic system. The Hamiltonian for the case be written as

$$H = \sqrt{p^2 + m_q^2} + \sqrt{p^2 + m_Q^2} + V(r) \quad (1)$$

Where m_q and m_Q are the light and heavy quark mass respectively, p is the relative momentum of each quark, $V(r)$ is the confined part of the quark-antiquark potential. Here, we consider a general power potential with a colour Coulomb term of the form [5, 8]

$$V(r) = \frac{-\alpha_c}{r} + Ar^\nu \quad (2)$$

where $\alpha_c = \frac{4}{3}\alpha_s$, α_s being the strong running coupling constant, A and ν are the potential parameters. We employ the trial wave function,

$$R(r) = 2\sqrt{\frac{\mu^3}{\sqrt{\pi}}} e^{-\mu^2 r^2/2} \quad (3)$$

and use the virial theorem, to get spin average mass from the hamiltonian defined by Eqn.(1). Here μ is the variational parameter. The parameters used

here are $m_{u/d} = 0.36 \text{ GeV}$, $m_s = 0.4 \text{ GeV}$, $m_b = 4.2 \text{ GeV}$, and $\alpha_c = 0.36$ (for B meson) and $\alpha_c = 0.33$ (for B_s meson). The remanning model parameter A is fixed for each choices of ν so as to get the experimental ground state spin average masses of B and B_s mesons. The spin average masses of $B - B^*$ and the $B_s - B_s^*$ mesons are computed using the experimental values of $M_B = 5.279 \text{ GeV}$, $M_{B^*} = 5.325 \text{ GeV}$, $M_{B_s} = 5.369 \text{ GeV}$, $M_{B_s^*} = 5.417 \text{ GeV}$ respectively [3]. The behavior of potential strength A with power index ν is shown in Fig 1. It is found to have an exponential behavior with ν . For computing the mass difference between $B - B^*$ and the $B_s - B_s^*$ mesonic states, we consider the spin dependent part of the usual OGEP given by [9]

$$V_{SS}(r) = \frac{4}{3} \frac{\pi \alpha_c}{m_Q m_{\bar{q}}} \left[S(S+1) - \frac{3}{2} \right] \delta^{(3)}(\vec{r}) \quad (4)$$

The computed results of S-state of B and B_s with different choices of $\nu = 0.5$ to 2.0 are listed in Table-1 along the existing experimental as well as with other theoretical model predictions.

3 The Decay constants of the neutral open beauty mesons

The decay constants of mesons are important parameters in the study of leptonic or non-leptonic weak decay processes and in the neutral $B - \bar{B}$ mixing process. In the nonrelativistic limit, the decay constant can be expressed through the ground state wave function at the origin $\psi_P(0)$ by the Van-Royen-Weisskopf formula [14]. Though most of the models predict the mesonic mass spectrum successfully, there are disagreements in the predictions of their decay constants [15]. So, we reexamine the predictions of the decay constants with different choices of potential index ν . We consider the nonrelativistic expression for f_P as [14]

$$f_P^2 = \frac{3 |R_P(0)|^2}{\pi M_P} \quad (5)$$

The results computed for B and B_s mesons are tabulated in Table-2 along with other theoretical model predictions.

4 Mixing Mass Parameter of $B_q(q \in d, s)$ mesons

In the standard model, the transitions $B_q^0 - \bar{B}_q^0$ and $\bar{B}_q^0 - B_q^0$ are due to the weak interaction. The netural B_d and B_s mesons mix with their antiparticles

Table 1: Mass of $B_q(q \in d, s)$ mesons

| Meson | ν | μ GeV | $ R(0) ^2$ GeV ³ | M_P in GeV | | M_V in GeV | |
|-------|-------|--------------|--------------------------------|--------------|-----------|--------------|-----------|
| | | | | Our | Others | Our | Others |
| B_d | 0.5 | 0.450 | 0.206 | 5.291 | | 5.324 | |
| | 0.7 | 0.472 | 0.237 | 5.287 | | 5.325 | |
| | 0.9 | 0.487 | 0.261 | 5.285 | | 5.327 | |
| | 1.0 | 0.493 | 0.271 | 5.284 | 5.279[3] | 5.327 | 5.324[3] |
| | 1.1 | 0.499 | 0.280 | 5.283 | 5.279[28] | 5.327 | 5.324[28] |
| | 1.3 | 0.508 | 0.295 | 5.282 | 5.285[10] | 5.329 | 5.324[10] |
| | 1.5 | 0.515 | 0.308 | 5.279 | | 5.328 | |
| | 1.7 | 0.522 | 0.320 | 5.278 | | 5.329 | |
| | 1.9 | 0.527 | 0.330 | 5.277 | | 5.329 | |
| | 2.0 | 0.529 | 0.334 | 5.276 | | 5.329 | |
| B_s | 0.5 | 0.491 | 0.267 | 5.378 | | 5.413 | |
| | 0.7 | 0.515 | 0.308 | 5.374 | | 5.414 | |
| | 0.9 | 0.532 | 0.339 | 5.371 | | 5.415 | |
| | 1.0 | 0.539 | 0.353 | 5.370 | 5.369[3] | 5.416 | 5.416[3] |
| | 1.1 | 0.545 | 0.365 | 5.369 | 5.373[28] | 5.416 | 5.421[28] |
| | 1.3 | 0.555 | 0.385 | 5.365 | 5.375[10] | 5.416 | 5.412[10] |
| | 1.5 | 0.563 | 0.403 | 5.365 | | 5.418 | |
| | 1.7 | 0.570 | 0.418 | 5.364 | | 5.418 | |
| | 1.9 | 0.576 | 0.431 | 5.363 | | 5.419 | |
| | 2.0 | 0.578 | 0.436 | 5.362 | | 5.419 | |

Table 2: Decay constant, Mixing Mass Parameter and lifetime of $B_q (q \in d, s)$ mesons

| Meson | ν | f_P in MeV | | Δm_q in ps^{-1} | | Lifetime τ in ps | |
|-------|-------|--------------|---------------------------|---------------------------|---------|-------------------------|-----------------|
| | | Our | Others | Our | Others | Our | Others |
| B_d | 0.5 | 192 | | 0.45 | | 1.55 | |
| | 0.7 | 206 | | 0.52 | | 1.56 | |
| | 0.9 | 216 | | 0.57 | | 1.56 | |
| | 1.0 | 220 | 196(29)[22] | 0.59 | 0.50[3] | 1.57 | 1.530 |
| | 1.1 | 223 | 189 [7] | 0.61 | | 1.57 | ± 0.009 [3] |
| | 1.3 | 230 | $190(7)_{-1}^{+24}$ [23] | 0.65 | | 1.57 | |
| | 1.5 | 235 | 203(23) [24] | 0.68 | | 1.57 | |
| | 1.7 | 239 | 206(20)[26] | 0.70 | | 1.58 | |
| | 1.9 | 243 | 210 (9)[25] | 0.72 | | 1.58 | |
| | 2.0 | 244 | | 0.73 | | 1.58 | |
| B_s | 0.5 | 217 | | 17.67 | | 1.43 | |
| | 0.7 | 233 | | 20.35 | | 1.43 | |
| | 0.9 | 244 | | 22.40 | | 1.44 | |
| | 1.0 | 249 | 216[22] | 23.26 | 17[3] | 1.44 | 1.466 |
| | 1.1 | 253 | 218 [7] | 24.04 | | 1.44 | ± 0.059 [3] |
| | 1.3 | 260 | $217(6)_{-28}^{+32}$ [23] | 25.38 | | 1.45 | |
| | 1.5 | 266 | 236(30) [24] | 26.52 | | 1.45 | |
| | 1.7 | 271 | 236 ± 30 [26] | 27.49 | | 1.45 | |
| | 1.9 | 275 | 244 (21)[25] | 28.33 | | 1.45 | |
| | 2.0 | 277 | | 28.71 | | 1.45 | |

leading to oscillations between the mass eigenstates. This mass oscillation is parameterized as the mixing mass parameter given by [12]

$$\Delta m_B = \frac{G_F^2 m_t^2 M_{B_q} f_{B_q}^2}{6\pi^2} g(x_t) \eta_t |V_{tq}^* V_{tb}|^2 B \quad (6)$$

where $\eta_t \approx 0.55$ is the glunoic correction to the oscillation [11], B is the bag parameter and its value is taken from the lattice result as 1.34 [12], while the pseudoscalar mass (M_{B_q}) and the pseudoscalar decay constant (f_{B_q}) of the beauty mesons are taken as our spectroscopic parameters determined using potential models. The $g(x_t)$ factor is the Inami-Lim function given by

$$g(x_t) = \frac{1}{4} + \frac{9}{4(1-x_t)} - \frac{3}{2(1-x_t)^2} - \frac{3}{2} \frac{x_t^2}{(1-x_t)^3} \ln x_t \quad (7)$$

Here, $x_t = \frac{m_t^2}{M_W^2}$. The values of m_t (174 GeV), M_W (80.403 GeV) and the CKM matrix elements $V_{tb}(1)$, $V_{td}(7.4 \times 10^{-3})$ and $V_{ts}(40.6 \times 10^{-3})$ are taken from the Particle Data Group [3]. The results are also tabulated in Table-2.

5 The Lifetime of the neutral open beauty mesons

We compute the lifetime of these neutral $B_q(q \in d, s)$ mesons by computing their semileptonic decay widths and by using their experimental branching ratios. The inclusive semileptonic decay width of the open beauty flavour mesons are computed using the expression given by

$$\Gamma(B \rightarrow l \bar{\nu}_l X_c) = \frac{G_F^2 m_b^5}{192 \pi^3} |V_{cb}|^2 [f(x) - \frac{\alpha_s}{\pi} g(x)] \quad (8)$$

Here, m_b the mass of b quark. Generally, it is taken as the model mass parameter coming from the fitting of its mass spectrum. Within the potential confinement scheme, we consider it as the effective mass of the quarks, m_q^{eff} [27]. Thus, effective b quark mass (m_b^{eff}) is defined as

$$m_b^{eff} = m_b \left(1 + \frac{E_{bind}}{m_b + m_{\bar{q}}} \right) \quad (9)$$

to account for its bound state effects. The binding effect has been calculated as $E_{bind} = M_{bq} - (m_b + m_q)$, where m_b and m_q are the model mass parameters employed in its spectroscopic study and M_{bq} is the mass of the mesonic state. The effective mass of the quarks would be different from the adhoc choices of the model mass parameters. For example, within the meson the mass of the

quarks may get modified due to its binding interactions with other quark. Thus, the effective mass of the b quark will be different when it is in $b\bar{s}$ combinations or in $b\bar{d}$ combinations due to the residual strong interaction effects of the bound systems. The functions $f(x)$ and $g(x)$ appeared in Eqn. 8 correspond to the phase space correction and the QCD correction at the bc vertex in this decay. They are computed from [18, 19], where the parameter x is computed as $x = \left(\frac{m_c}{m_b^{eff}}\right)^2$. The lifetime of B_d and B_s mesons are computed from the relation $BR(B_q \rightarrow l\bar{\nu}_l X_c) = \Gamma(B_q \rightarrow l\bar{\nu}_l X_c)\tau_{B_q}$. The results are listed in Table-1 with other theoretical and experimental values for comparisons.

6 Results and Discussion

We have employed the coulomb plus power potential form to study the the mixing mass parameters and lifetimes of the B_d and B_s mesons using the spectroscopic ground state parameters. Here we solved the Schrödinger equation using variational approach [5, 6]. Our potential parameters are fixed with respect to the center of weight ground state $1S$ mass of the B_q mesons. In Table-2, we tabulate our predictions for the f_p and Δm_q and lifetime of the neutral open beauty mesons. Our results are compared with the available experimental / other theoretical values. The prediction for Δm_q and lifetime of the B_q mesons are very close to the experimentally observed values. It can be seen that the mixing mass parameter is more sensitive to the choice of the potential index while the life time is least sensitive to the potential choice. In conclusion we find the overall predictions in accordance with the experimental values in the range of potential index $0.5 \leq \nu \leq 0.7$. Thus the present study is an attempt to indicate the importance of spectroscopic parameters (*i.e.* masses and decay constants) in the weak decay processes.

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